O.R. Applications

Heuristics for sourcing from multiple suppliers with alternative quantity discounts

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Abstract

In this paper, we analyze the impact of supplier pricing schemes and supplier capacity limitations on the optimal sourcing policy for a single firm. We consider the situation where the total quantity to be procured for a single period is known by the firm and communicated to the supplier set. In response to this communication, each supplier quotes a price and a capacity limit in terms of a maximum quantity that can be supplied to the buyer. Based on this information, the buyer makes a quantity allocation decision among the suppliers and corresponding to this decision is the choice of a subset of suppliers who will receive an order. Based on industry observations, a variety of supplier pricing schemes from the constituent group of suppliers are analyzed, including linear discounts, incremental units discounts, and all units discounts. Given the complexity of the optimization problem for certain types of pricing schemes, heuristic solution methodologies are developed to identify a quantity allocation decision for the firm. Through an extensive computational comparison, we find that these heuristics generate near-optimal solutions very quickly. Data from a major office products retailer is used to illustrate the resulting sourcing strategies given different pricing schemes and capacity limitations of suppliers in this industry. We find for the case of capacity constrained suppliers, the optimal quantity allocations for two complex pricing schemes (linear discount, and incremental units discount) are such that at most one selected supplier will receive an order quantity that is less than its capacity.

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1. Introduction

Consider the following procurement process followed by a major office products retailer with headquarters in Florida, USA.

The centralized purchasing organization (CPO) for the retailer is responsible for the procurement of all commodity type products which are sold

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through retail outlets. Orders for these types of products are placed by the CPO on a periodic basis and in general, a four step process characterizes the procurement process. First, for each time period, the CPO aggregates the total estimated requirements for each product based on input from the retail outlets. Second, using a web-based interface (which is set-up to allow access only to prequalified suppliers), the CPO posts timing and quantity requirements for the commodity product(s). Third, suppliers respond by quoting prices and quantity limitations, if any, for delivering the products to the central warehouse maintained by the CPO. Finally, a CPO planner analyzes the supplier submitted information and allocates requirements to suppliers.

The final step of this process for supplier selection and order placement was biased towards selecting a single supplier who could supply the entire set of requirements rather than an explicit focus on costs quoted by the suppliers. The primary motivation for such a strategy (i.e., single sourcing) was that it was easier to manage order receipts from one supplier at the central warehouse. In a more recent analysis of supplier responses to the CPO posting of requirements and timing information, it was found that suppliers were starting to quote more complex discount pricing schemes and thus, the supplier selection and order placement decision was becoming increasingly difficult.

This scenario motivates this paper and our focus is on analyzing the supplier selection and order placement decisions in the presence of alternative supplier pricing schemes and supplier capacity limitations. It is well known that sourcing decisions in this setting are extremely complex and also require frequent reassessment. For example, suppliers often offer discount schedules to induce larger purchases by offering progressively lower unit prices for progressively larger purchase quantities. If a single product is available from many qualified vendors, identifying the optimal selected supplier set and corresponding quantity allocations is a difficult decision. A further complicating issue is that decisions must often be made quickly and with limited information due to time pressures (Rubin and Benton, 1993). This fast paced decision environment is not always conducive to determining optimal (i.e., cost minimizing) solutions. Therefore, heuristic procedures that produce optimal or near-optimal feasible solutions are of significant value for decision makers.

More specifically, this paper examines supplier selection and quantity allocation decisions for the acquisition of a firm’s total requirements for a single product from a pool of suppliers offering a variety of pricing schedules. The specific supplier pricing schemes examined are linear quantity discount, incremental quantity discount, and an all-unit quantity discount. Heuristic methods are proposed for the case of linear discount, incremental and all units discount pricing schemes. Data obtained from a major office products retailer is used to further illustrate the application these approaches.

This paper is organized as follows. In the next section, we review the relevant prior literature and in Section 3, we develop our general sourcing model. Due to the complexities associated with obtaining optimal solutions to the model formulated, we propose heuristics for obtaining feasible solutions very quickly and in Section 4, we analyze the performance of these heuristics through an extensive numerical analysis. This is followed by an application of our heuristics to data obtained from an office products retailer in Section 5, and finally, the implications and conclusions of our research are discussed in Section 6.

2. Relevant literature

Order quantity or lot sizing decisions can be largely influenced by alternative supplier pricing schemes. In prior research, the most common pricing schemes that have been assumed are the constant price, the all unit discounted price, and the incremental unit discounted price. Another variation of a discounted pricing scheme which is commonly used in the marketing literature is that of a linear discounted price where the price linearly decreases with increases in the order quantity (Jeu-land and Shugan, 1985). The motivation for using discounted pricing schemes stems from the fact that it tends to encourage buyers to procure larger quantities and to reap operating advantages (such as economies of scale) for the buyer. From a coordination perspective, it has been shown that both the buyer and the supplier can realize higher overall profits if discounting schemes are used to set transfer prices (Wang, 2005).

There is a substantial amount of literature related to the quantity discount problem. From an economics perspective, Buchanen (1953) and Garbor (1955)
both provide the motivation for using quantity discounts. Porteus (1971) examines the incremental units quantity discount problem with a piecewise concave increasing cost function. Rather than reviewing all of the substantial work on quantity discounts, we refer the reader to Benton and Park (1996), Munson and Rosenblatt (2001), and Dolan (1987) for excellent overviews. Our focus here will be on analyzing the prior research which is most closely related to our work addressing the buyer’s optimal sourcing strategy given a pool of several suppliers.

Abad (1988a,b) incorporates price dependent demand into lot-sizing models with alternate supply acquisition schemes for a single supplier. Optimal lot size and selling price are simultaneously solved under linear and constant price-elastic demand. A supplier offers an all-unit quantity discount in Abad (1988a) and an incremental quantity discount in (1988b). An iterative procedure is developed to handle the lot-size and selling price interdependency. Burwell et al. (1997) extend the work of Abad by developing an optimal lot-sizing and selling price algorithm for a single item given that the supplier offered quantity and freight discounts and there is price dependent downstream demand. Numerical results indicate that the maximized profit with an all units discount scheme dominates the maximized profits under incremental discounts.

Benton (1991) uses Lagrangian relaxation to evaluate a purchasing manager’s resource constrained order quantity decisions given alternative pricing schemes from multiple suppliers. The author assumes that the decision maker has a limited budget and storage space for ten items offered by three vendors, each quoting three all units discount levels for each item. The objective is to minimize total acquisition and inventory costs. The manager must choose a single supplier for all items. However, if multiple sourcing is allowed, the optimal objective is 8% lower than the single source optimum. Rubin and Benton (1993) use Lagrangian relaxation to formulate a separable dual problem. A branch and bound algorithm with a best bound branching rule is developed to close any duality gap. To allow multiple sourcing among the multiple items, a merged discount schedule is constructed which quotes the lowest price among all suppliers for each quantity of each item. As an extension to this work, Rubin and Benton (2003) analyze the same purchasing scenario, except that suppliers quote incremental quantity discount schedules instead of all units. A solution methodology similar to their 1993 paper is employed.

Another Lagrangian relaxation based heuristic is developed by Guder et al. (1994) to solve a buyer’s multiple item material cost minimization problem with incremental discounts offered by a single supplier. The buyer has a single resource constraint and demand for items are independent. The complexity of the problem lies in evaluating all feasible price level sequences. For large problems with $n$ items and $m$ price breaks, the optimal price level sequence can be obtained in $O(nm)$. For 100 items with 8 price breaks each, numerical experiments result in precise solutions quickly. Their heuristic is adapted from Pirkul and Aras (1985) which analyzed the all units discount version of this problem.

Chauhaun and Proth (2003) start by analyzing a single product purchasing problem for a manufacturer who attempts to source a fixed quantity from multiple suppliers. Each supplier quotes a unique pricing scheme which integrates a fixed cost plus a concave increasing cost in the quantity sourced. Further, if an order is placed with a supplier, it must be within a minimum and maximum range. After characterizing a result for an optimal solution to this problem, they develop a heuristic which generates optimal solutions for all of the 20 test-problems generated by them. They conclude their paper by extending their analysis to the case of multiple suppliers and multiple buyers.

In the context of this paper, the modeling of supplier pricing schemes using a fixed setup cost plus a concave increasing component does not adequately capture the alternative pricing schemes that are prevalent not only in the operations literature but also those used in practice. Thus, the primary difference in our work as compared to the prior research is that we analyze the buyer’s problem under more common supplier pricing schemes. In the next section, we develop our modeling approach and characterize each of these pricing schemes.

3. Sourcing model

3.1. Preliminaries

Our analysis of the supplier sourcing decisions focuses on a single product, single period model of a system consisting of $N$ suppliers ($i = 1, \ldots, N$) and a single buying firm. Based on observed industry practices, we assume the following three-stage sequential decision framework for our analysis. At
the first stage, the buying firm communicates the total quantity of the single product (Q) which it will procure from the suppliers. Following this, in the next stage, each supplier i discloses a pricing scheme and a related maximum quantity that it can provide to the firm (yi). After receiving this information, the firm makes the supplier sourcing decision (qi) for each supplier in the third stage.

We model the problem for a buying firm which is either a channel intermediary with a fixed quantity contract from a set of downstream firms or a manufacturer making procurement quantity decisions using automated materials planning systems (such as MRP). In both cases it is realistic to assume that the buying firm can declare with reasonable certainty the total quantity Q to be procured from the suppliers. As noted earlier, zero fixed ordering costs for the buying firm are being assumed in line with our motivating example of the office products retailer.

There are three additional factors that need to be clarified in the context of our analysis. First, each supplier is an independent operator and hence, there are no opportunities for supplier collusion/collaboration in our setting. Second, the pricing scheme disclosed by each supplier is all inclusive and includes the logistics/transportation cost. Finally, supply lead times are assumed to be relatively constant and thus, are not incorporated in our analysis. In the next section, we describe the alternative supplier pricing schemes (and in some cases, the related supplier capacity) parameters which are investigated in this paper.

3.2. Supplier pricing schemes and capacity

There are several types of supplier pricing schemes which have been documented and analyzed in prior research. These schemes are primary drivers for analyzing the sourcing decisions since they have a direct impact on firm-level profits. Further, supplier capacity also can drive some of these pricing schemes. The three distinct supplier pricing schemes and capacity which we incorporate in this paper are:

1. Linear discount price

Under this scheme, each supplier i discloses a linearly declining per unit price in the quantity qi defined as \( a_i - b_i q_i \) (\( a_i, b_i > 0 \)) within the capacity range \([0, y_i]\). We also assume that the linearly discounted price is such that \( a_i - b_i y_i > 0 \) for each supplier i (i.e., the buyer is always charged a positive price for any quantity procured from supplier i).

2. Incremental units discounted price

Under this arrangement, each supplier i discloses the traditional incremental units discounting scheme (Nahmias, 2001) which is dependent on the quantity \( q_i \) purchased from supplier i. To specify such a scheme, we first define \( k = 1, \ldots, K_i \) as the index for discount classes offered by supplier i. Corresponding to each discount class \( k \) for a supplier i, define \([l_{ik}, u_{ik}]\) as the minimum and maximum quantities (such that \( u_{ik} = y_i \)), and \( w_{ik} \) as the per unit price. It is assumed that \( w_{i1} > w_{i2} > \cdots > w_{iK} \) for each supplier i.

3. All units discount price

Under this arrangement, each supplier i discloses the traditional all units discounting scheme (Nahmias, 2001) which is dependent on the quantity \( q_i \) purchased from supplier i. As with the incremental units scheme, let \( k = 1, \ldots, K_i \) as the index for discount classes offered by supplier i. Corresponding to each discount class \( k \) for a supplier i, define \([l_{ik}, u_{ik}]\) as the minimum and maximum quantities (such that \( u_{ik} = y_i \)) and \( v_{ik} \) as the per unit price. It is assumed that \( v_{i1} > v_{i2} > \cdots > v_{iK} \) for each supplier i.

Next, we formulate the specific sourcing model for each pricing scenario and provide analytical/experimental insights into the sourcing strategy under each case.

4. Analysis

4.1. Linear discount price

The sourcing model for this case can be formulated as

\[
\text{Minimize} \quad Z_L = \sum_{i=1}^{n} (a_i - b_i q_i)(q_i) \tag{1}
\]

subject to:

\[
\sum_{i=1}^{n} q_i = Q, \tag{2}
\]

\[
0 \leq q_i \leq y_i \quad \forall i. \tag{3}
\]

This is a concave minimization problem with \( n \) decision variables and \( n + 1 \) constraints. For the case where each supplier has adequate capacity (i.e., \( y_i \geq Q \ \forall i \)) to meet the aggregate requirement \( Q \), it is obvious that the firm will choose to source the
We update the requirements for the firm based on this phase of our procedure considers all suppliers in this non-decreasing order of the total costs assuming that the procedure, we start by ranking suppliers in this sequential manner. If the supplier capacity is less than the firm requirements, we place an order for the maximum quantity the supplier can deliver. Then if the supplier capacity is also show that for special cases (such as equal supplier capacities and for the case when the supplier pricing functions are Lipshitz continuous with the directional derivates following a specific ordered relationship), a polynomial time algorithm can be used to optimally solve the problem. Our approach in obtaining a solution to the problem exploits the fact that the following general result characterizes an optimal solution to this problem.

**Result 1.** There exists at least one optimal solution to our sourcing model such that \( q_i = 0 \) or \( q_i = y_i \) for all \( i \) suppliers except that there may be at most one supplier \( j \) for which \( 0 < q_j < y_j \).

**Proof.** See Appendix A. \( \Box \)

The heuristic we propose builds upon this result and is structured as follows. In the first phase of the procedure, we start by ranking suppliers in non-decreasing order of the total costs assuming that each supplier is given an order for the maximum he/she can supply or firm’s requirements. The next phase of our procedure considers all suppliers in this list in a sequential manner. If the supplier capacity is less than the firm requirements, we place an order for the maximum quantity the supplier can deliver. Then we update the requirements for the firm based on this allocation, and again rank the remaining suppliers in non-decreasing order of total costs and the process repeats itself until the remaining requirements are zero. In the final phase of the procedure, we consider switching the partial order quantity among all suppliers who have been allocated a positive order quantity. Details of our heuristic are as follows:

1. Define the active supplier set, \( \Omega \), as consisting of all suppliers. Also, set \( Q = Q \).
2. For each supplier \( i \) included in \( \Omega \), compute \( r_i = a_i - b_i(\min\{Q', y_i\}) \).
3. Rank suppliers in increasing order of \( r_i \) and index suppliers in this ranked list \([1], [2], \ldots, [N] \).
4. Set \( j = 1 \).
5. Set \( d[j] = \min\{Q', y[j]\} \). Remove supplier \( j \) from \( \Omega \).
6. Set \( Q' = Q' - q[j] \). If \( Q' > 0 \) go to 2, else go to 7.
7. For all suppliers \( k \), with \( q_k > 0 \), explore all possible improvements in the solution by switching the partial order quantity within this selected supplier set.
8. Store the best solution as Solution A.
9. Repeat the above process, except at step 2, calculate \( r_i = \min\{Q', y_i\} + (a_i - b_i(\min\{Q', y_i\})) \), and in step 6, store the best solution as Solution B.
10. Choose solution A or Solution B based on the better objective function value.

To evaluate the solution quality of this heuristic, we carried out numerical experiments by randomly generating 30 test problems. For every problem, the supply base size, \( N \), was set to 10, and the total requirement, \( Q \), equals 2000 units. Further, for each supplier \( i \), the pricing and capacity parameters were randomly generated as follows: base price \( (a_i) \) from a discrete uniform distribution with parameters \([0, 200]\); price elasticity \( (b_i) \) from a uniform distribution with parameters \([0, 1]\); and capacity \( (y_i) \) from a discrete uniform distribution with parameters \([0, 1000]\). For each supplier the parameters were generated such that \( a_i - b_i y_i > 0 \).

The results of this evaluation are presented in Table 1. The gap reported in this table is the percentage deviation of the heuristic solution cost from the optimal solution cost. As can be seen, in 21 of the 30 problems, the heuristic solution was optimal. Based on the other 9 problems, the worst case heuristic solution gap was 5.78\%, while the average gap was 0.59\%. The optimal solution was obtained using LINGO (release 9.0) with the Global Solver Capability (available from www.lindo.com) on a Pentium 4, 3.6 GHz PC with 1 GB RAM. For each instance we solved a problem with 10 decision variables and 11 constraints. This optimal solution was obtained within 2 seconds for 29 of the 30 problems and within 1 second for the remaining problem while the heuristic solution was obtained within 1 second for all 30 problems.
Table 1
Linear discount heuristic performance

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<th>Instance</th>
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<th>Optimal</th>
<th>Gap %</th>
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Mean gap % 0.59 Worst case gap % 5.78

difficult. Further, determining the optimal solution also requires the use of a commercial software package (in this case, LINGO) and hence, there might be some cost advantages for using the heuristic which does provide “good” quality solutions very quickly. Finally, as previously mentioned, many companies have limited the number of suppliers under consideration in the past due to the complexity of these problems. In the future, we believe that companies such as the office product retailer discussed in the paper can consider simultaneous bids from a larger supplier set by utilizing the heuristic.

4.2. Incremental units discount price

The sourcing model for this case can be formulated as

\[
\text{minimize } Z_{IU} = \sum_{i=1}^{n} \sum_{k=1}^{K_i} \sum_{j=1}^{k-1} (w_{ij}(u_{ij} - u_{ij-1}) + \frac{w_{ik}(q_{ik} - l_{ik})y_{ik}}{C_0})
\]

subject to:

\[
\sum_{i=1}^{n} \sum_{k=1}^{K_i} q_{ik} = Q,
\]

\[
\sum_{k=1}^{K_i} y_{ik} \leq 1 \quad \forall i,
\]

\[
l_{ik}y_{ik} \leq q_{ik} \leq u_{ik}y_{ik} \quad \forall i, k,
\]

\[
y_{ik} \in [0, 1].
\]

This is non-linear, integer formulation with \([2\sum_{i=1}^{n}K_i]\) decision variables and \([2\sum_{i=1}^{n}K_i]+(n+1)\) constraints. For the case where each supplier discloses a pricing scheme such that \(u_{ik} \geq Q \forall i\), to meet the aggregate requirement \(Q\), it is trivial to show that the firm will choose to source the complete requirement \(Q\) from supplier \(j\) such that \(w_{jk} = \min_{1 \leq j \leq n \leq K_i} \{w_{jk} | l_{ik} \leq Q \leq u_{ik}\}\). In line with the prior pricing schemes, the single sourcing strategy is an optimal choice for this situation.

As with the linear discount pricing scheme, the dominance of the single sourcing strategy is questionable when every supplier does not have adequate capacity to meet the aggregate requirements \(Q\) and in fact, given the concavity of the objective, this problem is also NP-hard (Burke et al., 2006). In this case, again, we note that Result 1 stated previously for the linear discount pricing scheme still holds since our objective function is piecewise concave in \(q_i\). Defining \(f_{ik}(q_{ik}) = \sum_{j=1}^{k-1} w_{ij}(u_{ij} - u_{ij-1}) + w_{ik}(q_{ik} - l_{ik})\) where \(l_{ik} \leq q_{ik} \leq u_{ik}\), we propose the following heuristic algorithm for obtaining solutions to the sourcing model under this pricing scenario.

1. Define the active supplier set, \(\Omega\), as consisting of all suppliers. Also, set \(Q' = Q\).
2. For each supplier \(i\) included in \(\Omega\),
   - If \(u_{ik} < Q'\), compute \(r_i = f_{ik}(u_{ik})/u_{ik}\), and set \(q_i = u_{ik}\).
   - otherwise,
     - DO \(k = 1, 2, \ldots, K_i\)
     - If \(l_{ik} \leq Q' \leq u_{ik}\), \(r_i = f_{ik}(Q')/Q'\) and \(q_i = Q'\).
     - END
3. Rank suppliers in increasing order of $r_i$ and index suppliers in this ranked list $[1], [2], \ldots, [N]$, and set $j = 1$.
4. Set $Q' = Q - q_{[j]}$. Remove supplier $j$ from $\Omega$. If $Q' > 0$ go to 2, else go to 5.
5. For all suppliers $k$, with $q_k > 0$, explore all possible improvements in the solution by switching the partial order quantity within this selected supplier set. 
6. Store the best solution as Solution A.
7. Repeat the above process, except at step 2, calculate in the first if statement $r_i = f_i(u_K)$, and in the DO loop $r_i = f_i(Q')$; store the best solution as Solution B.
8. Choose solution A or Solution B based on the better objective function value.

To evaluate the solution quality of this heuristic, we carried out numerical experiments by randomly generating 30 test problems. As with the linear discount pricing case, for every problem, the supply base size, $N$, was set to 10, and the total requirement, $Q$, equals 2000 units. The procedure used to generate the relevant parameters is as follows:

1. Generate the number of price breaks for each supplier $i$ ($K_i$) from a discrete uniform distribution with parameters $[0, 10]$;
2. Generate the unit price for the first price break ($w_{1i}$) from a uniform distribution with parameters $[0, 1]$;
3. Set the unit prices for each of the remaining price breaks ($k = 2, \ldots, K_i$) as $w_{ik} = w_{ik-1} - 0.05$;
4. Set the lower bound for the first price break ($l_{ik}$) to 0;
5. Generate the upper bound for the first price break ($u_{ik}$) from a discrete uniform distribution with parameters $[0, 100]$;
6. To generate the lower and upper bounds for the remaining price breaks ($k = 2, \ldots, K_i$), the following iterative process if followed:
   (a) Set $k = 2$.
   (b) Set $l_{ik} = u_{ik-1} + 1$.
   (c) Generate $u_{ik} = l_{ik} + U$ where $U$ is generated from a discrete uniform distribution with parameters $[0, 100]$.
   (d) If $k = K_i$ stop, else set $k = k + 1$ and go to (b).

The results of this evaluation are presented in Tables 2 and 3. In Table 2, the costs associated with the optimal and heuristic solutions, and the percentage deviation of the heuristic solution from the optimal solution (Gap) for every problem is shown. As can be seen, in 27 of the 30 problems, the heuristic solution was optimal. Based on the remaining 3 problems, the worst case heuristic solution gap was 0.40%, while the average gap was 0.025%. The optimal solution was obtained using LINGO (release 9.0) with the Global Solver Capability (available from www.lindo.com) on a Pentium 4, 3.6 GHz PC with 1 GB RAM. Table 3 provides details of the computation times (in seconds) for obtaining the optimal solution as well as the sizes of the problems solved. For all 30 cases, the heuristic solution was obtained in less than one second.

<table>
<thead>
<tr>
<th>Instance</th>
<th>Heuristic</th>
<th>Optimal</th>
<th>Gap %</th>
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Mean gap % 0.025 Worst case gap % 0.40
while Table 3 shows that to obtain the optimal solution, the computation times range from a low of 2 seconds to a high of 282 seconds. Based on this, it is quite clear that the heuristic is valuable for solving these type of quantity discount problems since it provides comparable quality solutions in significantly less time.

4.3. All units discount price

The sourcing model for this case can be formulated as

\[
\begin{align*}
\text{Minimize} \quad Z_{AU} &= \sum_{i=1}^{n} \sum_{k=1}^{K_i} v_{ik} q_{ik} y_{ik} \\
\text{subject to:} \quad \sum_{i=1}^{n} \sum_{k=1}^{K_i} q_{ik} &= Q, \\
& \sum_{k=1}^{K_i} y_{ik} \leq 1 \quad \forall i, \\
& l_{ik} y_{ik} \leq q_{ik} \leq u_{ik} y_{ik} \quad \forall i, k, \\
& y_{ik} \in [0, 1].
\end{align*}
\]

This is a non-linear, integer formulation with \(2n + 2\) decision variables and \(2n + 1\) constraints. For the case where each supplier has adequate capacity (i.e., \(u_{ik} \geq Q \forall i\)) to meet the aggregate requirement \(Q\), it is trivial to show that the firm will choose to source the complete requirement \(Q\) from a single supplier \(j\) such that \(V_j^P = \min_{1 \leq j \leq n} \{ V_i^P\} \). In this case, \(V_j^P\) is the price per unit offered by supplier \(i\) for quantity \(Q\).

Again, the dominance of the single sourcing strategy is questionable when every supplier does not have adequate capacity to meet the aggregate requirements \(Q\). Further, in this case, our objective is discontinuous and thus, the result stated for the prior two cases (for a continuous concave objective function) do not necessarily apply. On the other hand, the property stated in the result is quite easy to incorporate in a heuristic and hence, the following heuristic was proposed to generate feasible solutions to our problem.

1. Define the active supplier set, \(Q\), as consisting of all suppliers. Also, set \(Q = Q_0\).
2. For each supplier \(i\) included in \(Q\),
   - If \(u_{ik} < Q\), compute \(r_i = v_{ik}\) and set \(q_i = u_{ik}\),
   - otherwise,
     - DO \(k = 1, 2, \ldots, K_i\)
     - If \(l_{ik} \leq Q \leq u_{ik}\), \(r_i = v_{ik}\) and \(q_i = Q\)
     - END
     - compute \(r_i = \max\{v_{ik}, v_j|l_{ij} \leq Q \leq u_{ij}\}\).
3. Rank suppliers in increasing order of \(r_i\), and index suppliers in this ranked list \([1], [2], \ldots, [N]\), and set \(j = 1\).
4. Set \(Q = Q - q_{[j]}\). Remove supplier \(j\) from \(Q\). If \(Q > 0\) go to 2, else go to 5.
5. For all suppliers \(k\), with \(q_k > 0\), explore all possible improvements in the solution by switching the partial order quantity within this selected supplier set.
6. Store the best solution as Solution A. Repeat the above process, except at step 2, calculate in the first if statement \(r_i = v_{ik} u_{ik}\), and in the DO loop \(r_i = v_{ik} Q\); store the best solution as Solution B.

Table 3

<table>
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<th>Instance</th>
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<th>Problem size</th>
<th>Instance</th>
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</table>
7. Choose solution A or Solution B based on the better objective function value.

To evaluate the solution quality of this heuristic, we carried out numerical experiments by randomly generating 30 test problems. The parameter settings for each problem are identical to those generated for the incremental units discount price case except that for each supplier $i$, the unit price $v_{ik} = w_{ik}$ for $k = 1, \ldots, K_i$.

The results of this evaluation are presented in Table 4. The gap reported in this table is the percentage deviation of the heuristic solution cost from the optimal solution cost obtained using LINGO. As can be seen, in 17 of 30 problems, the heuristic solution was optimal. The worst case heuristic solution gap was 1.58%, while the average gap was 0.12%. The optimal solution was obtained using LINGO (release 9.0) with the Global Solver Capability (available from www.lindo.com) on a Pentium 4, 3.6 GHz PC with 1 GB RAM. For each instance, the problem size solved is identical to that reported in for the case of incremental discounts as shown in Table 3. Under this pricing scheme, the optimal solutions were obtained within at most 3 seconds of run time while the heuristic solution was obtained within 1 second for all 30 problems. As with the linear discount case, we observe that the optimal solution can be obtained fairly quickly for this setting and hence, the usefulness of the heuristic is questionable. However, similar reasons as those discussed earlier motivate the use of the heuristic since it does provide “good” quality solutions very quickly.

4.4. Summary

Based on our analysis, we can offer the following general conclusions:

- Sourcing strategy: For any pricing scheme, the only time a single supplier sourcing strategy is preferred is when the lowest cost supplier has adequate capacity to meet the entire demand for the firm. In all other cases, a multiple sourcing strategy is the general choice.
- Supplier quantity allocations: When suppliers quote quantity discount pricing schemes, the optimal quantity allocations are more difficult to determine since the problem is NP-hard. However, the heuristics we propose for solving these problems are such that we can obtain near-optimal solutions very quickly. More specifically, for the incremental discount case, a stronger case can be made for using the proposed heuristic method.

In addition, we ran supplementary experiments to investigate the implications of problem size on the computational times and heuristic performance. In general, the problem size can be measured by calculating the number of decision variables and the number of constraints for each discount scheme. For the case of linear discounts, the number of variables is

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<th>Gap %</th>
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Mean gap % 0.12 Worst case gap % 1.58
simply equal to the number of suppliers \((n)\), whereas the number of constraints is equal to \(n + 1\). For the case of incremental and all units discounts, the number of decision variables and the number of constraints is shown in the following equation:

\[
\text{Number of decision variables} = 2 \sum_{i=1}^{n} K_i
\]

\[
\text{Number of constraints} = 2 \sum_{i=1}^{n} K_i + n + 1
\]

Qualitatively, the results are very similar to those shown for the numerical examples shown in Tables 1–4. Specifically, the heuristic solution quality is very accurate with the average gap being close to 0.0%. Furthermore, the computational savings are the greatest for the incremental units discount scheme where the average time to compute the optimal solution is the highest. We now turn to an application of our heuristic approaches to data obtained from an office products distributor.

5. Application

The Central Purchasing Organization (CPO) of a major office products retailer is frequently faced with complex sourcing decisions due to the presence of quantity discount schemes embedded within various suppliers’ bids. We obtained data from the CPO for two commodity products (Product A and Product B) among hundreds of commodity stock keeping units (SKU) that must be frequently replenished for retail sale. The firm’s current total requirement is to purchase 9855 units of Product A and 7680 units of Product B. The portfolio of supplier bids for Product A consists of six suppliers, four with constant price quotes and two with quantity discount pricing while Product B’s bid portfolio consists of eight suppliers, four with constant price quotes and four with discount pricing. All suppliers in both portfolios have also specified their capacity limitations in units. Tables 5 and 6 contain bid information for Products A and B, respectively.

Using this data, the heuristics proposed in this paper can be applied to identify the supplier set as well as the corresponding supplier quantity allocations. In order to do so, however, we make the following assumptions/modifications:

- Information on whether the quantity discount pricing schemes were an all units discount or an incremental units discount was not available.

Thus, we create two quantity allocation problems for each product. The first problem assumes price discounts are incrementally applied to order quantities, the second assumes price discounts are for all units ordered. Based on this, for each product, we also apply the all units discount and the incremental units discount heuristics.

- It is also of interest to assess the solution quality of the heuristic methods. In order to do so, we randomly generate nine additional feasible total requirements \((Q)\) for each product and based on this, we apply the heuristics to ten different problems for each product and quantity discount scheme. As before, the optimal solution for each problem is obtained using LINGO (release 9.0).
with the Global Solver Capability (available from www.lindo.com) on a Pentium 4, 3.6 GHz PC with 1 GB RAM.

In Tables 7 and 8, we report the results for the Product A and Product B, respectively. Observations based on these results are:

- **Product A (Table 7)**
  - In order to obtain the optimal solution, an integer non-linear model with 18 decision variables and 25 constraints was formulated.
  - In all 10 problems under the assumption of the incremental units discount scheme, the heuristic solution is optimal. Given that the heuristic computing time was less than 1 second for all problems while the solution time to obtain the optimal solution ranged from a low of 1 second to a high of 9 seconds, the heuristic method provides “good” solutions very quickly.
  - In 7 of the 10 problems under the assumption of the all units discount scheme, the heuristic solution is optimal while in the other 3 problems the gap is at most 0.53%. In this context, the solution time for the heuristic solution was again less than 1 second for each problem, while the optimal solution was obtained in at most 2 seconds. As with our earlier results, it seems that the value of the heuristic for this pricing scheme is more when we need to solve such problems repeatedly with little reliance on commercial software packages.

- **Product B (Table 8)**
  - In order to obtain the optimal solution, an integer non-linear model with 36 decision variables and 45 constraints was formulated.
  - In all 10 problems under the assumption of the incremental units discount scheme, the heuristic solution is optimal. Given that the heuristic computing time was less than 1 second for all problems while the solution time to obtain the optimal solution ranged from a low of 3 seconds to a high of 31 seconds, the heuristic method provides “good” solutions very quickly.

### Table 7
Comparison of solutions for product A

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<th>All units discount</th>
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### Table 8
Comparison of solutions for product B

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In 9 of the 10 problems under the assumption of the all units discount scheme, the heuristic solution is optimal while in the other 1 problem the gap is 0.03%. In this context, the solution time for the heuristic solution was again less than 1 second for each problem, while the optimal solution was obtained in at most 2 seconds. As with our earlier results, it seems that the value of the heuristic for this pricing scheme is more when we need to solve such problems repeatedly with little reliance on commercial software packages.

In sum, the results of this application provide additional validity for the usefulness of the heuristics developed for the discount pricing schemes proposed in this paper.

6. Conclusions

The analysis of alternate supplier base pricing schemes in this paper provides guidance for a buying firm’s optimal sourcing strategy. Given a total order quantity \( Q \) that the buying firm must procure, a deterministic model is analyzed which highlights the importance of supplier capacity on the buying firm’s sourcing decision. Specifically, if all of the suppliers in the base possess enough capacity to individually provide \( Q \) units, then it is optimal to single source from the least cost supplier evaluated at \( Q \) units. The case when the single supplier sourcing strategy is not optimal is when the minimum cost supplier does not have adequate capacity to fill an entire order.

In cases where the suppliers’ capacity is individually inadequate for a buying firm’s product requirements, a firm’s sourcing problem can become extremely complex. Therefore, identifying an optimal solution may be resource (time) prohibitive for supply chain sourcing professionals. In such situations, the heuristics developed in this paper can be expected to efficiently provide good quality solutions. In summary, the heuristic solutions to randomly generated test problems arrived at the optimal solution in 72% of the instances. Furthermore, the average optimality gap for the ninety test problems is 0.21%. Using data from a retail office products firm, we are also able to validate the claim that our heuristics provide near-optimal solutions to the multiple supplier sourcing problem. Further, these heuristics can be used on a repetitive basis to generate real-time solutions in terms of supplier quantity allocations.

The results of this paper highlight the importance of supplier scale on the buying firm’s sourcing decision. While the model introduced here focuses on the buying firm’s optimal decision, future research is necessary to further investigate the supplier’s quotation decision as well. For example, consider the situation where a supplier has sufficient scale to meet the buying firm’s total order quantity but cannot offer the lowest price. Competitive models of supplier bidding using complex pricing schemes could address some of the behavioral aspects of pricing as well.

Appendix 1. Proof of Result 1

Result 1. There exists at least one optimal solution to our sourcing model such that \( q_i = 0 \) or \( q_i = y_i \) for all \( i \) suppliers except that there may be at most one supplier \( j \) for which \( 0 < q_j < y_j \).

Proof. Recall that our problem is

\[
\text{Min } Z = \sum_{i=1}^{n} g_i(q_i)
\]

s.t.

\[
\sum_{i=1}^{n} q_i = Q, \quad 0 \leq q_i \leq y_i \quad \forall i,
\]

where \( g_i(q_i) = a_i q_i - (b_i q_i^2) \).

This is obviously a concave cost minimization problem for supply of single product. As such, the proof of this result is quite similar to that of Chauhaun and Proth (2003, pp. 375–376).

Let \( S^1 = \{q_1^1, q_2^1, \ldots, q_n^1\} \) be a feasible solution to the above problem. Assume that there exists \( i, j \in \{1, 2, \ldots, n\} \) such that:

\[
0 < q_i^1 < y_i \quad \text{and} \quad 0 < q_j^1 < y_j
\]

and that

\[
\frac{\partial g_i}{\partial q_i}(q_i^1) \leq \frac{\partial g_j}{\partial q_j}(q_j^1)
\]

choose \( \delta = \text{Min}\{y_i - q_i^1, q_j^1\} \)

Set:

\[
q_i^2 = q_i^1 + \delta, \quad q_j^2 = q_j^1 - \delta.
\]
Obtain a new feasible solution, \( S^2 \), by replacing \( q_i^1 \) with \( q_i^2 \) and \( q_j^1 \) with \( q_j^2 \).

Adding (15) and (16):

\[
q_i^2 + q_j^2 = q_i^1 + q_j^1.
\]

This together with the definition of \( \delta \), insure that \( S^2 \) is feasible.

Now, since relation (14) holds for all feasible solutions we know that:

\[
\frac{\partial g_i}{\partial q_i}(q_i^i) \leq \frac{\partial g_j}{\partial q_j}(q_j^j)
\]

and for any \( q_i^i \geq q_i^1 \) and \( q_j^j \leq q_j^1 \), we know that:

\[
g_i(q_i^i) - g_i(q_i^1) \leq g_j(q_j^j) - g_j(q_j^1).
\]

So

\[
g_i(q_i^2) - g_i(q_i^1) \leq g_j(q_j^2) - g_j(q_j^1),
\]

and, therefore, \( \sum_{i=1}^{n} g_i(q_i^2) \leq \sum_{i=1}^{n} g_i(q_i^1) \).

Repeating this process will lead to a solution \( S^2 \) that verifies the conditions for Result 1.

References


