Information-Sensitive Inventory Management when Records are Inaccurate

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Nicole DeHoratius, Adam Mersereau, & Linus Schrage.  
**Retail Inventory Management when Records are Inaccurate.**  
Forthcoming in *M&SOM*.

Adam Mersereau.  
**Information-Sensitive Replenishment when Inventory Records are Inaccurate.**  
Nearly finished.
Retail Inventory Record Inaccuracy

- **Inventory record inaccuracy**: Discrepancy between recorded inventory and what is actually on a retailer’s shelf.

- **Causes** (*DeHoratius and Raman, 2007*)
  - Customer theft
  - Shoplifting
  - Damage to merchandise
  - Replenishment errors
  - Imperfect audits
  - Etc.
Data from Gamma (DeHoratius and Raman, 2007)
What to do?

1. **Prevention:** Reduce incidence of record inaccuracy.
   - Process quality and conformance.
   - Improved tracking technology (e.g., RFID)

2. **Correction:** Perform inventory audits.

3. **Integration:** Accept there is record inaccuracy and use decision tools that account for it.
How does record inaccuracy impact optimal replenishment in a lost sales environment?

Talk Outline:

1. Managing Record Inaccuracy using a Bayesian Inventory Record
2. Optimal Inventory Policies for Short Horizons
3. An Approximate POMDP Algorithm for Longer Horizons
4. Summary and Research Directions
Related Work

Inventory Management with Record Inaccuracy
- Kang and Gershwin (2005)
- Kök and Shang (2007)
- Lee and Özer (2007)
- Bensoussan, Cakanyildirim, Sethi (2007)

Inventory Management with Demand Learning
- Lariviere and Porteus (1999)
- Ding, Puterman, Bisi (2002)
- Chen and Plambeck (2007)
- Lu, Song, Zhu (2007)
Assumptions

- Single SKU.
- Periodic review.
- Unobserved lost sales.
- Perfect replenishment process with lead time 0.
- Unobserved daily inventory perturbation: “invisible” demand.
- Arbitrary (discrete) demand distributions.
- Customer demands and “invisible” demands independent.
Daily Sequence of Events

0. Initial inventory $I_{t-1}$.
1. Replenishment $R_t$ arrives.
2. Demand $D_t$ arrives, observe sales $S_t = \min\{D_t, I_{t-1} + R_t\}$.
3. Place replenishment order $R_{t+1}$.

Information state $\phi_t = \{I_0, R_0, \ldots, R_t, S_0, \ldots, S_t\}$.

Inventory dynamics:
- Physical inventory: $I_t = I_{t-1} + R_t - S_t$.
- Recorded inventory: $J_t = J_{t-1} + R_t - S_t$. 
Daily Sequence of Events

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Adam Mersereau
Information-Sensitive Inventory Management 10
Daily Sequence of Events

1. Initial inventory $I_{t-1}$.
2. Replenishment $R_t$ arrives.
3. Demand $D_t$ arrives, observe sales $S_t = \min\{D_t, I_{t-1} + R_t\}$.
4. Invisible demand $V_t$ arrives, unobserved invisible “sales” $U_t = \min\{V_t, I_{t-1} + R_t - S_t\}$.
5. Place replenishment order $R_{t+1}$.

Information state $\phi_t = \{I_0, R_0, \ldots, R_t, S_0, \ldots, S_t\}$.

Inventory dynamics:
- Physical inventory: $I_t = I_{t-1} + R_t - S_t - U_t$.
- Recorded inventory: $J_t = J_{t-1} + R_t - S_t$. 
Illustrated Daily Sequence of Events

- Physical inventory:

\[ I_{t-1} \rightarrow R_t \rightarrow I_t \]

- Recorded inventory:

\[ J_{t-1} \rightarrow R_t \rightarrow J_t \]

- Bayesian inventory record (BIR):

\[ P_{t-1} \rightarrow R_t \rightarrow P_t \]
**Bayesian Inventory Record (BIR)**

- Maintain a probabilistic belief of inventory level, updated using Bayes rule.
  \[ P_t(i) \equiv \Pr \{ I_t = i | \phi_t \} \]

- Bayesian updating reflects:
  - If see no sales, then actual inventory might be zero.
  - If see sales, then actual inventory could not have been zero.
Updating $P_t(\cdot)$: An Example

| $\Pr (I_{t-1} + R_t)$ | $S_t$ | $\Pr (I_{t-1} + R_t|S_t)$ | $P_t(\cdot) = \Pr (I_t)$ |
|------------------------|-------|-----------------------------|-----------------------------|
| ![Histogram](image1)   | 0     | ![Histogram](image2)       | ![Histogram](image3)       |
| ![Histogram](image4)   | 1     | ![Histogram](image5)       | ![Histogram](image6)       |
| ![Histogram](image7)   | 2     | ![Histogram](image8)       | ![Histogram](image9)       |
| ![Histogram](image10)  | 3     | ![Histogram](image11)      | ![Histogram](image12)      |

$(D_t \sim \text{NegBin}(1.0, 0.5), V_t$ difference of two Poissons with means 0.2.)
Critical fractile-based myopic replenishment heuristic.

Audit heuristic based on Expected Value of Perfect Information (EVPI).

Methods for calibrating model parameters using retailer audit data.

Proof that $P_t(\cdot)$-based policies avoid persistent “freezing.”

Simulation study with sensitivity analysis.
Sales/Inventory Tradeoff

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<th>Observed in-stock probability</th>
<th>Average physical inventory held</th>
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</table>

Target CF = 99%

(—– approx. 90% confidence intervals —–)

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Information-Sensitive Inventory Management
How does record inaccuracy impact optimal replenishment in a lost sales environment?

Optimal decisions

- Require solving partially observed Markov decision problem (POMDP) with belief state $P_t(\cdot)$.
- Intractable due to “curse of dimensionality.”

Our approach

- Characterize replenishment for a two-period special case.
- Numerically examine an approximate POMDP policy for more general problems.
Effects of Record Inaccuracy on Replenishment

Myopic and forward-looking replenishment:

**Uncertainty effect:** *Stock more* to buffer additional uncertainty brought by record inaccuracy.

**Direct Loss effect:** *Stock less* so there is less to lose through theft and damage.

Forward-looking replenishment:

**Persistent Inventory effect:** *Stock less* to reduce the possibility of carrying too much inventory into next period.

**Information effect:** *Stock less* to improve information in BIR.

- Perishable goods with unknown demand distribution, unobserved lost sales.
- Forward-looking policy stocks more than myopic policy.
- **Information effect:** Stock more to reduce stockouts, see better demand data.
Inspiration: Replenishment with Demand Learning


- Perishable goods with unknown demand distribution, unobserved lost sales.
- Forward-looking policy stocks more than myopic policy.
- **Information effect:** Stock more to reduce stockouts, see better demand data.

Chen and Plambeck (2007), Lu, Song, and Zhu (2007)

- Non-perishable goods with unknown demand distribution, unobserved lost sales.
- **Persistent Inventory effect:** Stock more may become stock less to avoid carrying too much inventory into next period.
Initial stock $I_0$. 
One-period Problem

0. Initial stock $I_0$.

1. Place initial order $R_0$.

   $\rightarrow$ cost $c$ per unit.
One-period Problem

0 Initial stock $I_0$.

1 Place initial order $R_0$.
   \[ \rightarrow \text{cost} \ c \ \text{per unit}. \]

2 Demand $D_0$ arrives, yielding visible sales $S_0$.
   \[ \rightarrow \text{cost} \ p \ \text{per unit unsatisfied}. \]
One-period Problem

0. Initial stock $I_0$.

1. Place initial order $R_0$.
   \[ \rightarrow \text{cost } c \text{ per unit.} \]

2. Demand $D_0$ arrives, yielding visible sales $S_0$.
   \[ \rightarrow \text{cost } p \text{ per unit unsatisfied.} \]

3. Salvage
   \[ \rightarrow \text{value } (c - h) \text{ per unit left over.} \]
One-period Problem

If \( I_0 = i_0 \) known, order
\[
R_{0,1}^{noV}(\delta_{i_0}) = \min \left\{ R_0 \geq 0 : \Pi_{R_0+i_0} \geq \frac{p-c}{p+h-c} \right\}, \text{ where } D_0 \sim \Pi.
\]

If \( I_0 \) is unknown with distribution \( P_0 \), order
\[
R_{0,1}^{noV}(P_0) = \min \left\{ R_0 \geq 0 : W_{R_0} \geq \frac{p-c}{p+h-c} \right\}, \text{ where } D_0 - I_0 \sim W.
\]

When \( E[I_0|P_0] = i_0 \), expect \( R_{0,1}^{noV}(P_0) \geq R_{0,1}^{noV}(\delta_{i_0}) \) for \( \frac{p-c}{p+h-c} \) sufficiently large.
One-period Problem

If $I_0 = i_0$ known, order
$$R_{0,1}^{\text{noV}}(\delta_{i_0}) = \min \left\{ R_0 \geq 0 : \Pi_{R_0+i_0} \geq \frac{p-c}{p+h-c} \right\}, \text{ where } D_0 \sim \Pi.$$

If $I_0$ is unknown with distribution $P_0$, order
$$R_{0,1}^{\text{noV}}(P_0) = \min \left\{ R_0 \geq 0 : W_{R_0} \geq \frac{p-c}{p+h-c} \right\}, \text{ where } D_0 - I_0 \sim W.$$

When $E[I_0|P_0] = i_0$, expect $R_{0,1}^{\text{noV}}(P_0) \geq R_{0,1}^{\text{noV}}(\delta_{i_0})$ for $\frac{p-c}{p+h-c}$ sufficiently large.

Uncertainty Effect
With $I_0$ uncertainty, stock more to buffer added uncertainty.
One-period Problem with Invisible Demand

0. Initial stock $I_0$.

1. Place initial order $R_0$.
   → cost $c$ per unit.

2. Demand $D_0$ arrives, yielding visible sales $S_0$.
   → cost $p$ per unit unsatisfied.

3. Invisible demand $V_0$, yielding invisible “sales” $U_0$.

4. Salvage
   → value $(c - h)$ per unit left over.

**Theorem:** Optimal order $R_{0,1}^*(P_0) \leq R_{0,1}^{noV}(P_0)$ for any $P_0$, assuming $c - h \geq 0$. 

Direct Loss Effect

With invisible demand, stock less so there is less to lose to shrinkage.
One-period Problem with Invisible Demand

0. Initial stock $I_0$.

1. Place initial order $R_0$.
   \[\rightarrow\] cost $c$ per unit.

2. Demand $D_0$ arrives, yielding visible sales $S_0$.
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**Direct Loss Effect**

With invisible demand, *stock less* so there is less to lose to shrinkage.
Two-period Problem

0 Initial stock $I_0$.

1 Place initial order $R_0$.
   $\rightarrow$ cost $c$ per unit.

2 Demand $D_0$ arrives, yielding visible sales $S_0$.
   $\rightarrow$ cost $p$ per unit unsatisfied.

3 Invisible demand $V_0$, yielding invisible “sales” $U_0$.

4 Salvage
   $\rightarrow$ value $(c - h)$ per unit left over.
Two-period Problem

0 Initial stock $I_0$.

1 Place initial order $R_0$.
   $\rightarrow$ cost $c$ per unit.

2 Demand $D_0$ arrives, yielding visible sales $S_0$.
   $\rightarrow$ cost $p$ per unit unsatisfied.

3 Invisible demand $V_0$, yielding invisible “sales” $U_0$.

6 Salvage
   $\rightarrow$ value $(c - h)$ per unit left over.
Two-period Problem

0 Initial stock $I_0$.

1 Place initial order $R_0$.
   $\rightarrow$ cost $c$ per unit.

2 Demand $D_0$ arrives, yielding visible sales $S_0$.
   $\rightarrow$ cost $p$ per unit unsatisfied.

3 Invisible demand $V_0$, yielding invisible “sales” $U_0$.
   $\rightarrow$ cost $h$ per unit left over.

4 Place replenishment order $R_1$.
   $\rightarrow$ cost $c$ per unit.

5 Demand $D_1$ arrives, yielding visible sales $S_1$.
   $\rightarrow$ cost $p$ per unit unsatisfied.

6 Salvage
   $\rightarrow$ value $(c - h)$ per unit left over.

(Myopic decision optimal when $I_0$ known and no $V_0$.)
Two-period Problem

0. Initial stock $I_0$.

1. Place initial order $R_0$.
   \[ \rightarrow \text{cost } c \text{ per unit.} \]

2. Demand $D_0$ arrives, yielding visible sales $S_0$.
   \[ \rightarrow \text{cost } p \text{ per unit unsatisfied.} \]

3. Invisible demand $V_0$, yielding invisible "sales" $U_0$.
   \[ \rightarrow \text{cost } h \text{ per unit left over.} \]

4. Place replenishment order $R_1$.
   \[ \rightarrow \text{cost } c \text{ per unit.} \]

5. Demand $D_1$ arrives, yielding visible sales $S_1$.
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6. Salvage
   \[ \rightarrow \text{value } (c - h) \text{ per unit left over.} \]

(Myopic decision optimal when $I_0$ known and no $V_0$.)
Let $R_{0,2}^*(\delta_{i_0}) =$ optimal initial replenishment in 2-period problem with $I_0 = i_0$ known.

**Proposition:** $R_{0,2}^*(\delta_{i_0}) \leq R_{0,1}^*(\delta_{i_0})$

- $\exists$ examples where inequality is strict.
- Relies on $I_0$ known.
- Proof sketch:
  - Conditional on $D_0$, suppose $R_0 \rightarrow R_1$.
    Then $R_0 + 1 \rightarrow R_1 - 1$ or $R_0 + 1 \rightarrow R_1$.
  - Characterize period 1 cost difference by considering these two cases.
  - Show period 1 cost non-decreasing as a function of $R_0$. 
Let $R_{0,2}^{\text{perish}}(\delta_{i_0}) = \text{optimal initial replenishment if we allow } R_1 \text{ negative}. \text{ Can show}$

$$R^*_0,2(\delta_{i_0}) \leq R_{0,2}^{\text{perish}}(\delta_{i_0}) \leq R^*_0,1(\delta_{i_0})$$
Let $R_{0,2}^{\text{perish}}(\delta_{i_0}) = \text{optimal initial replenishment if we allow } R_1 \text{ negative. Can show}$

$$R_{0,2}^*(\delta_{i_0}) \leq R_{0,2}^{\text{perish}}(\delta_{i_0}) \leq R_{0,1}^*(\delta_{i_0})$$
Intuition

Let \( R_{0,2}^{perish}(\delta_{i_0}) \) = optimal initial replenishment if we allow \( R_1 \) negative. Can show

\[
\underbrace{R_{0,2}^*(\delta_{i_0}) \leq R_{0,2}^{perish}(\delta_{i_0}) \leq R_{0,1}^*(\delta_{i_0})}_{\text{Persistent Inventory effect}} \quad \underbrace{R_{0,2}^{perish}(\delta_{i_0})}_{\text{Information effect}}
\]
Intuition

Let $R_{0,2}^{\text{perish}}(\delta_{i_0}) = \text{optimal initial replenishment if we allow } R_1 \text{ negative.}$ Can show

$$R_{0,2}^*(\delta_{i_0}) \leq R_{0,2}^{\text{perish}}(\delta_{i_0}) \leq R_{0,1}^*(\delta_{i_0})$$

Persistent Inventory effect  Information effect

Persistent Inventory effect

- **Stock less** to reduce possibility of carrying too much inventory into next period.
- Arises from constraint $R_1 \geq 0$. 

Adam Mersereau  Information-Sensitive Inventory Management 37
Let $R_{0,2}^{\text{perish}}(\delta_{i_0}) = \text{optimal initial replenishment if we allow } R_1 \text{ negative. Can show}$

$$R_{0,2}^*(\delta_{i_0}) \leq R_{0,2}^{\text{perish}}(\delta_{i_0}) \leq R_{0,1}^*(\delta_{i_0})$$

- **Persistent Inventory effect**
  - Stock less to reduce possibility of carrying too much inventory into next period.
  - Arises from constraint $R_1 \geq 0$.

- **Information effect**
  - Stock less to improve knowledge about inventory level.
  - Arises from impact of $R_0$ on shape of $P_1(\cdot)$. 
Does “information effect” always cause manager *stock less*?
Does “information effect” always cause manager stock less?

**No.** Can choose cost parameters, demand distribution, $P_0$ to give a counterexample.
Does “information effect” always cause manager stock less?

- **No.** Can choose cost parameters, demand distribution, $P_0$ to give a counterexample.

- **Usually.** Found $R_{0,2}^*(P_0) \leq R_{0,2}^{perish}(P_0) \leq R_{0,1}^*(P_0)$ for each of 10000 randomly generated problems.
Remaining questions:
- Do effects persist for longer horizons?
- What are the magnitudes of the effects?
- How do effects vary with system parameters?

Would like to numerically compare:
- $R_{0,1}^{\text{noV}}$: Myopic policy ignoring invisible demand
- $R_{0,1}^{\ast}$: True myopic policy
- $R_{0,T}^{\ast}$: Optimal policy (Intractable to compute!)
Exact POMDP:
\[ J_t(P_t) = \min_{R_t \geq 0} \{ m(P_t, R_t) + E[J_{t+1}(B(P_t|R_t, S_t))] \} \]

Grid Interpolation Approach (Hauskrecht, 2000):
1. Maintain a finite set (grid) of template distributions.
2. Solve reduced dynamic program among the templates.
3. Approximate values of new beliefs by interpolating among templates.

Notes:
- Requires procedures for interpolation, template-generation.
- Overcomes curse of dimensionality.
- Provides a policy, lower bound on optimal value.
Numerical Results

- 10-period horizon.
- Visible demand: Negative binomial distribution, mean 2.0, variance 3.8.
- Invisible demand: Difference of two Poisson distributions, mean 0, variance 1.2.
- $c = 3$, $p = 9$, $h = 1$, $c_s = c - h = 2$.

<table>
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<th>Policy</th>
<th>Total Cost</th>
<th>Total Ordered</th>
<th>Average Stock</th>
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<td>101.36</td>
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<td>4.59</td>
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<tr>
<td>$R_{0,1}^{*}$</td>
<td>101.02</td>
<td>19.62</td>
<td>4.42</td>
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<tr>
<td>A-POMDP</td>
<td>100.69</td>
<td>19.00</td>
<td>4.13</td>
</tr>
</tbody>
</table>
Evolution of BIR

![Graphs showing expected value and standard deviation over time for different strategies: Myopic noV, True myopic, and A-POMDP.](image)

- **Expected Value**
  - Myopic noV
  - True myopic
  - A-POMDP

- **Standard Deviation**
  - Myopic noV
  - True myopic
  - A-POMDP
As Horizon Becomes Longer

### Problem Horizon vs. Average Cost

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Myopic noV</th>
<th>True myopic</th>
<th>A−POMDP</th>
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<td>6</td>
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</table>

### Problem Horizon vs. Average Stock

<table>
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<tr>
<th>Horizon</th>
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<th>True myopic</th>
<th>A−POMDP</th>
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Adam Mersereau
Information-Sensitive Inventory Management 46
As Invisible Demand Becomes More Variable

![Graphs showing the relationship between Invisible Demand Variance and Average Cost, Average Stock for Myopic noV, True myopic, and A–POMDP.](image)

- **Average Cost**
  - Myopic noV
  - True myopic
  - A–POMDP

- **Average Stock**
  - Myopic noV
  - True myopic
  - A–POMDP
Identified and isolated several effects of record inaccuracy on optimal replenishment
- Short term incentive to stock more to buffer added uncertainty.
- Long term incentive to stock less to rein in future uncertainty.

Approximate POMDP approach to partially observable inventory management
- Orders less, achieves lower cost than myopic policy.
- Effect of record inaccuracy increases with horizon, inaccuracy variance.
- Useful for other assumptions, decisions.
- Impact of scheduled audits on replenishment.
- Characterizing optimal joint audit/replenishment policies.
- Multiple SKUs with substitution.
- Integration of partially observed inventory management, parameter estimation.